Rate of Period Change as a Diagnostic of Cepheid Properties

David G. Turner
Department of Astronomy and Physics, Saint Mary's University, Halifax, Nova Scotia B3H 3C3, Canada
turner@crux.smu.ca

Mohamed Abdel-Sabour Abdel-Latif
Department of Astronomy and Astrophysics, National Research Institute Of Astronomy and Geophysics (NRIAG), Box 11242, Helwan, Cairo, Egypt
sabour2000@hotmail.com

and

Leonid N. Berdnikov
Sternberg Astronomical Institute and Isaac Newton Institute of Chile, Moscow Branch, 13 Universitetskij prosp., Moscow 119899, Russia
berdnik@sai.msu.ru

ABSTRACT

Rate of period change $\dot{P}$ for a Cepheid is shown to be a parameter that is capable of indicating the instability strip crossing mode for individual objects, and, in conjunction with light amplitude, likely location within the instability strip. Observed rates of period change in over 200 Milky Way Cepheids are demonstrated to be in general agreement with predictions from stellar evolutionary models, although the sample also displays features that are inconsistent with some published models and indicative of the importance of additional factors not fully incorporated in models to date.

Subject headings: stars: Cepheids—stars: evolution

1. Introduction

Cepheids represent a brief phase in the post-main-sequence evolution of stars that originally had masses in excess of $\sim 3\frac{1}{2}M_{\odot}$ (Turner 1996). As evolved objects they populate the Cepheid instability strip in the HR diagram according to the manner in which they generate energy, depending upon strip crossing. Intermediate mass stars can become unstable to radial pulsation during shell hydrogen burning (first crossing), twice during core helium burning (second and third crossings), and according to some evolutionary models (Iben 1965; Becker et al. 1977; Becker 1985; Xu & Li 2004), twice during shell helium burning (fourth and fifth crossings). A common feature of many recent evolutionary models for intermediate-mass stars, which incorporate the new opacity tables (e.g., Meynet & Maeder 2000, 2002; Bono et al. 2000; Salasnich et al. 2000), is that they permit only three strip crossings for such stars, since core oxygen ignition occurs prior to a separate shell helium-burning phase.

In all cases the evolution of stars through the Cepheid instability strip should be associated with gradual changes in overall dimensions, and hence periods of pulsation: period increases for evolution towards the cool edge of the instability strip as the stellar radius grows, and period decreases for evolution towards the hot edge as the stellar radius decreases. The observed parabolic trends

in Cepheid O–C diagrams (plots of the differences between Observed times of light maximum and those Computed from a linear ephemeris) have been recognized for the past half century as evidence for the evolution of such stars through the instability strip (Paranego 1958; Struve 1959; Erleksova & Irkaev 1982). As noted by Struve (1959), "It appears that studies of period change are by far the most sensitive test available to the astronomer for detecting minute alterations in the physical characteristics of a star."

Observations of period changes in Cepheids have been matched with some confidence to evolutionary models of massive stars in various crossings of the instability strip (e.g., Turner 1998; Turner & Berdnikov 2001, 2004) in order to identify the direction of strip crossing for individual variables. When used for such purposes, the study of Cepheid period changes becomes an important tool for the characterization of individual members of the class.

In principle it should also be possible to use rate of period change for individual Cepheids to establish likely location within the instability strip. Because strip crossings for individual Cepheids occur at different rates and at different luminosities for specific stellar masses, the observed rates of period change must be closely related to strip crossing mode and location within the instability strip. Potential constraints are imposed by variations in chemical composition and pulsation mode, e.g., fundamental mode, first overtone, etc. (Berdnikov et al. 1997; Turner et al. 1999), as well as by our limited ability to establish small rates of period change for O–C data containing sizeable observational uncertainties (Szabados 1983). In this paper we demonstrate the link in more detail.

2. Basis of the Relationship

The link between rate of period change in Cepheids and location within the instability strip is illustrated with the aid of Fig. 1. The diagram is a theoretical HR diagram that depicts the location of the Cepheid instability strip according to the parameters derived for Milky Way Cepheids (Turner 2001), along with Geneva evolutionary tracks for stars of 4, 5, 7, and 10 $M_{\odot}$ at $Z = 0.008$ from Lejeune & Schaerer (2001). Lines of constant stellar radius are shown crossing various portions of the instability strip. According to the well established Cepheid period-radius relation, they should represent lines of constant pulsation period for individual Cepheids.

¿From an examination of Fig. 1 it is clear that, if one considers only Cepheids of a specific period and in a common crossing of the instability strip, those on the hot edge of the strip must be $\sim 20\%$ more massive than those on the cool edge of the strip. Since rate of evolution increases in proportion to the mass of a star, Cepheids lying on the hot edge of the strip are evolving faster, and hence changing their pulsation periods at a more rapid rate, than Cepheids of identical period lying on the cool edge of the strip. Rate of period change therefore relates directly to location within the instability strip for individual Cepheids. Differences in strip crossing modes are only a minor concern. Cepheids with increasing periods must be in the first, third, or fifth crossing of the strip, whereas Cepheids with decreasing periods must be in the second or fourth crossing of the strip.

A minor complication arises from restrictions on our ability to identify period changes in Cepheids tied solely to stellar evolution. Some Cepheids exhibit erratic period changes that appear to originate from random fluctuations in pulsation period. SZ Tau (Berdnikov & Pastukhova 1995), S Vul (Berdnikov 1994), and V1496 Aql (Berdnikov et al. 2004) are excellent examples, although in the first two cases it is possible to identify the underlying evolutionary modifications to pulsation period.

A study by Berdnikov & Ignatova (2000) may give the impression that stellar evolution has only a minor effect on Cepheid O–C diagrams, since it notes that parabolic trends were detected in only 67 of 230 Cepheids surveyed. That number is misleading, however, given that a previous survey by Turner (1998) had found parabolic trends in 137 Cepheids from a much smaller sample. It was actually intended to indicate the poor temporal coverage and lack of extensive O–C data available for many well-studied Galactic Cepheids, a situation that has been remedied in recent years by our ongoing program to obtain archival data on Cepheid brightness variations using the Harvard College Observatory Photographic Plate Collection. At present the parabolic trends in O–C diagrams typical of stellar evolution are found to be

extremely common. A survey by Glushkova et al. (2005) cites a typical frequency of $\sim 80\%$ in both cluster and field Cepheids, for example, although their "anomalous" objects include Cepheids like SV Vul in which the evolutionary trend is quite distinct (Turner & Berdnikov 2004). A more realistic frequency for Milky Way Cepheids displaying evolutionary trends is in excess of $\sim 90\%$. For many of the remaining objects, the evolutionary trends may be more obvious in longer time baselines of light curve coverage.

As also pointed out by Fernie (1990) and by Berdnikov & Turner (2004), the O–C trends indicative of evolution in Cepheids need not be strictly parabolic. If the rate at which a massive star is evolving through the instability strip is not constant with time, the O–C data for the associated Cepheid variable may be better described by a third or fourth order polynomial. The Cepheids Y Oph (Fernie 1990) and WZ Car (Berdnikov & Turner 2004) are two objects (of several hundred) where that appears to be the case. Such complications may affect the derived rates of period change, but in most cases only by small amounts. In the large majority of studies of Cepheid period changes, the derived rate of period change reflects the evolution of the star through the instability strip (see Szabados 1983).

3. Stellar Evolution Predictions

Most computational evolutionary models for evolved stars are used for constructing evolutionary tracks rather than testing for pulsation instability. But, as noted by citetpa58, it is possible to use the basic information they provide on gradual changes in luminosity and effective temperature to predict expected rates of period change for Cepheids of different period. A starting point is the well known period-density relation:

$$P\rho^{\frac{1}{2}} = \frac{PM^{\frac{1}{2}}}{(\frac{4}{3}\pi)^{\frac{1}{2}}R^{\frac{3}{2}}} = Q \ ,$$

where P is the pulsation period, ρ is the density, M is the stellar mass, R is the stellar radius, and Q, the pulsation constant, has a small period dependence (e.g., Kraft 1961; Fernie 1967) that we assume here varies as $P^{\frac{1}{8}}$ based upon an empirical analysis by Turner & Burke (2002). Differentiation of the period-density relation, in conjunction with the standard equation for stellar luminosity, therefore leads to the following result:

$$\frac{\dot{P}}{P} = \frac{6}{7}\frac{\dot{L}}{L} - \frac{24}{7}\frac{\dot{T}}{T} \ .$$

The desired quantity, the rate of period change $\dot{P}$, is obtained from tabulated differences in stellar luminosity and effective temperature as a function of age as a model star evolves through the instability strip.

For the present study we calculated values of $\dot{P}$ from the above relationship using computational stellar evolutionary models from a variety of available published sources, namely Maeder & Meynet (1988), Alibert et al. (1999), Lejeune & Schaerer (2001), and Claret (2004). The published data were used to compute different parameters, depending upon the availability of the necessary information. Alibert et al. (1999) cite parameters for stars of different mass reaching the hot and cool edges of the instability strip, so their data yield information only about rates of period change near the center of the strip. In other cases, such as Claret (2004), there is sufficient time resolution in the output parameters to track changes in pulsation period across individual instability strip crossings. For the remaining sources (Maeder & Meynet 1988; Lejeune & Schaerer 2001), including Claret (2004), we calculated rates of period change for the intersection of the evolutionary tracks with the observationally delineated boundaries of the instability strip defined empirically by Turner (2001), which are close to those predicted by models of pulsation instability (Alibert et al. 1999), as well as for points lying within the strip boundaries. Pulsation periods were established using the period-radius relation (Turner & Burke 2002). The present results differ from those obtained earlier (Turner & Berdnikov 2001, 2003) in being tied to a larger variety of models with a greater range of metallicity, and by the inclusion of a weak period dependence for Q in the period-density relation.

The computed results on rates of period change are plotted in Fig. 2 for all of the accessible models. Different symbols denote the different sources. Values calculated from the models of Alibert et al. (1999) are plotted using filled circles, while others are plotted using open circles. Plus signs indicate results calculated for stars evolving through

the hot and cool edges of the instability strip, with the rate of period change in general being larger on the hot edge of the instability strip, i.e., for more massive stars. Large symbols denote stars of solar metallicity, $Z = 0.02$, intermediate-sized symbols denote stars with metallicities of $Z = 0.01$ and $Z = 0.008$, and small symbols denote stars of very low metallicity, $Z = 0.001$ and $Z = 0.004$. Lines have been drawn to enclose those regions within which the results for different crossing modes appear to cluster. Sequences of points indicate models for which the time resolution was fine enough to calculate rate of period change over the entire crossing of the instability strip.

The distribution of data points in Fig. 2 suggests a variety of different conclusions regarding the models. First, the different models for the rapid first crossing of the instability strip are in very good agreement, and display very little variation with metallicity. The first crossing of the strip is a rapid transition for all stars, regardless of individual differences in rotation rate, etc., and that is evident from the models. Evidently the computational codes used for calculating the phases of shell hydrogen burning in stars, while perhaps differing in detail from one source to another, generate nearly identical results, the small variation in rate of period change at specific pulsation period arising from the finite width of the instability strip and the fact that more massive stars cross the strip at a greater luminosity and at a faster rate than less massive stars. For stars in the first crossing of the strip, high rate of period increase at specific pulsation period corresponds to stars on the hot edge of the strip, low rate of period increase to stars on the cool edge of the strip.

Negative period changes arise during the second crossing of the instability strip, which occurs during the blue loop phase of stellar evolution following the onset of core helium burning. The extent of the blue loop can depend upon a variety of factors (see, for example, Becker 1985; Xu & Li 2004), such as metallicity, the treatment of core overshooting, and the distribution of CNO elements throughout the star. All factors affect how far a star enters the instability strip during core helium burning, and presumably affects how rapidly it evolves within the strip. Given the potentially large differences in initial conditions for such stars as main-sequence objects, for example, large variations in initial rotation rate, one might expect real stars to display large variations in how far they penetrate the Cepheid instability stripm as core helium burning objects. Somewhat unexpectedly, there are also very large variations among the models stars as well.

Evidently, metallicity plays only a minor role in governing the *rate* at which stars traverse the instability strip. There is as much dependence on the specifics of the stellar evolutionary code used. The models of Alibert et al. (1999), for example, generate faster rates of period decrease than do other models, despite the use of common opacity tables. Models from individual sources are at least internally consistent in their predictions for stars of different masses and for stars in all portions of the second strip crossing. The rates of period decrease during individual strip crossings are also very similar to the variations predicted on the basis of mass differences, i.e, predicted variations in rate of period decrease at a specific pulsation period are generally small, except for long period Cepheids.

The third crossing of the instability strip occurs during the late stages of core helium burning, and gives rise to period increases, for which the predicted rates are depicted in the top portion of Fig. 2 along with those for the first crossing. Most of the comments regarding the second crossing of the strip apply equally to the third crossing. Again, metallicity seems to play a less important role in the predicted rates of period increase than differences in the evolutionary code. The models of Alibert et al. (1999) predict faster rates of period change (period increases in this case) than do other models, although with less consistency for stars of different mass. The rates of period increase during individual strip crossings are also similar to the variations predicted on the basis of mass differences, and predicted variations in the rate of period increase at a specific pulsation period are generally small.

A well-known problem arises for low-mass stars in the second and third crossings of the instability strip, since the blue loop phases of evolutionary models for stars of solar metallicity, $Z = 0.02$, do not enter the strip for $M < 4.75 M_{\odot}$ (see Alibert et al. 1999). Model stars of lower metallicity can traverse the strip at smaller masses, but often only on the cool edge. By inference,

most classical Cepheids of near-solar metallicity should have pulsation periods in excess of $\sim 3\frac{1}{2}$ days (e.g., Turner 1996), consistent with the observational sample. Nearby Milky Way Cepheids have abundances close to the solar values (e.g., Andrievsky et al. 2002a,b), and only a few have periods of less than $3\frac{1}{2}$ days. Many may be overtone pulsators.

The observational picture is illustrated in Fig. 3, which presents available data on period changes for over 200 Cepheids, as obtained from the literature (Berdnikov & Pastukhova 1994a,b, 1995; Berdnikov et al. 1997; Turner 1998; Berdnikov & Ignatova 2000; Berdnikov et al. 2003) and ongoing research studies by the authors (e.g., Berdnikov & Turner 2004; Berdnikov et al. 2004). The relationships plotted in Fig. 3 depict the regions within which the model calculations appear to cluster.

It has been pointed out previously (e.g., Szabados 1983; Fernie 1984; Turner 1998) that the observed rates of period change in Cepheids are generally a good match to predictions from stellar evolutionary models. The data of Fig. 3 provide further confirmation of that conclusion. Moreover, three further conclusions can be reached. First, once consideration is taken of the expected changes arising from evolution through the instability strip, the observed period changes in Cepheids are unlikely to contain any sizable component arising from another source. There are only a few exceptions to such a conclusion, and they are rather unusual objects like V1496 Aql (Berdnikov et al. 2004), which exhibits period changes dominated by random fluctuations in pulsation period.

Second, the observed period changes in Cepheids deviate in small but important ways from what is expected according to predictions based upon specific stellar evolutionary models. The models of Alibert et al. (1999), for example, predict faster second and third crossings of the strip than those observed, and at much different rates. In contrast, the observed period changes in Cepheids are very similar for objects likely to be in the second and third crossings. The models of Claret (2004) are most consistent with observations in that regard, but it is necessary to have a more complete mass grid of models constructed in the same manner to make a more detailed comparison.

Third, the range in observed rates of period change for most Cepheids is smaller than that resulting from a comparison of the results from different evolutionary models. That is somewhat surprising, given our previous discussion about potentially wide variations in initial conditions for Cepheid predecessors. Evidently real stars are similar enough in their internal characteristics that they evolve at fairly similar rates through the Cepheid instability strip.

The proportions of Cepheids in different crossing modes and in different period ranges in Fig. 3 are also reasonably consistent with evolutionary expectations. For example, stars in the first crossing of the instability strip during shell hydrogen burning are evolving about two orders of magnitude faster than stars in second and third crossings, so their relative numbers should be small. The two Cepheids in Fig. 3 undergoing large rates of period increase and falling in the predicted region for first crossers are Polaris (α UMi) and DX Gem. We assume that both are first crossers, as was also argued for Polaris by Turner et al. (2005). Moreover, the observed rate of period change for Polaris is now seen to be exactly what stellar evolutionary models predict for a star lying on the cool edge of the instability strip for first crossers.

The proportion of Cepheids with detectable parabolic trends in their O–C data also increases noticeably towards short pulsation periods, which is again consistent with the evolutionary expectation that the most abundant pulsators must be those evolving most slowly through the instability strip. There is a curious anomaly in the distribution of short period Cepheids, where essentially no variables are found to have rates of period change as predicted for stars in second and third crossings of the strip at $P \leq 3.5$ days ($\log P \leq 0.55$). Such stars have progenitor masses of less than $\sim 4\ M_\odot$ (Turner 1996), where stellar evolutionary models for solar metallicity stars predict that the evolutionary tracks for core helium burning stars should no longer enter the strip. The short period cutoff in the observational sample is therefore consistent with expectations from stellar evolutionary models. But the existence of stars of $P \leq 3.5$ days with rates of period change roughly an order of magnitude faster than predicted for stars in second and third crossings of the instability strip is not. The uncertainties in the observed rates of period change in the anomalous objects are generally

much too small to resolve the anomaly by invoking systematic errors in the values of $\dot{P}$.

An additional factor that can be important for short period Cepheids is overtone pulsation. The Cepheids in the observational sample have all been assumed to be fundamental mode pulsators, and require a displacement of +0.15 in log P to establish their proper locations in Fig. 3 if they are overtone pulsators. Yet the application of such corrections to all of the anomalous objects does not affect their distribution significantly; most still fall outside the region of $\dot{P}$-space predicted for stars in the second and third crossing of the instability strip. Current stellar evolutionary models are therefore unable to explain the existence of such stars, which suggests that the manner of treating the details of stellar evolution during blue loop stages is very important (see also Xu & Li 2004). That is one area where improvements to the observational sample on Cepheid period changes can play an important role in testing the results from stellar evolutionary models.

4. $\dot{P}$ as a Fundamental Parameter

In Fig. 3 the dispersion in the rates of period change $\dot{P}$ observed in long period Cepheids ($P > 10^{\rm d}$) is smaller than what is observed for the calculated dispersion in that parameter among different stellar evolutionary models. One might expect $\dot{P}$ to correlate closely with location in the instability strip for Cepheids in all strip crossings, according to the results of Fig. 1. It is informative to examine the observational data more closely to determine if that is the case.

As a first step, we note that the observed rates of Cepheid period change plotted in Fig. 3 fall mainly within specific bands delineated by linear margins of slope 3.0 separated by an order of magnitude range in $\dot{P}$. Fig. 4 is a separate plot of the data that displays such empirically defined margins. All but two of the long period Cepheids with increasing periods fall within the lower set of margins, as do the majority of short period Cepheids with increasing periods. Cepheids with decreasing periods display a greater dispersion in $\dot{P}$ that may be intrinsic, or may be caused by larger uncertainties in $\dot{P}$ for the stars, particularly those with small rates of period change.

The anomaly for Cepheids with $P \leq 3.5$ days (log $P \leq 0.55$) is again apparent in Fig. 4. All Cepheids of shorter period display faster rates of period change than is typical of variables populating the lower band, and there are a number of stars of longer period also falling in this region of rapid period change. Presumably those stars represent Cepheids in fourth and fifth crossings of the instability strip, with faster associated rates of period change. Multiple crossings of the instability strip appear to be possible for stars in late core helium burning stages, depending upon the CNO abundances in the hydrogen burning shells of such stars (Xu & Li 2004).

The finite range in stellar surface temperature for stars populating the instability strip at constant pulsation period implies distinct differences in pulsation efficiency that should coincide with marked differences in pulsation amplitude for Cepheids of similar period. On the hot edge of the strip the ionization zone is just beginning to reach depths where the piston mechanism for pulsation becomes efficient, so light amplitudes should be small but increasing with decreasing surface temperatures. On the cool edge the lower surface temperatures are associated with increased convective energy transport in the star's outer layers (Deupree 1980), so pulsation amplitudes should also be small.

The first study of Cepheid amplitudes as a function of position in the strip by Kraft (1963) was consistent with that picture, although small amplitude Cepheids were found only on the hot edge of the strip. All subsequent amplitude maps of the instability strip by Hofmeister (1967), Sandage & Tammann (1971), Payne-Gaposchkin (1974), Pel & Lub (1978), Turner (2001), and Sandage et al. (2004) have produced similar results, namely a sharp rise to maximum amplitude on the hot edge of the strip followed by a more gradual decline towards the cool edge.

Cepheid amplitudes display a period dependence as well as a dependence upon location within the strip, a natural consequence of an effect tied to surface gravity as well as pulsation efficiency. In order to eliminate that factor in characterizing Cepheid period changes, we have normalized the resulting values of blue light amplitude and $\dot{P}$ as follows: (i) blue amplitudes ΔB were standardized through the ratio $\Delta B/\Delta B(\max)$, where $\Delta B(\max)$ is the maximum value of ΔB for the

star's pulsation period, and (ii) $\dot{P}$ was adjusted to the equivalent value for a Cepheid with a pulsation period of 10^{d} using the empirically-obtained slope plotted in Fig. 4.

Fig. 5 plots such data for Cepheids with $12^{\mathrm{d}} \leq P \leq 40^{\mathrm{d}}$ and increasing pulsation periods ($P \simeq 20^{\mathrm{d}}$). The upper part of the diagram plots the individual data, while the middle part of the diagram plots running means for the data. The lower part of the diagram is an alternate interpretation of the same data, as described below. Similar plots are given in Fig. 6 for Cepheids with $4^{\mathrm{d}} \leq P \leq 8^{\mathrm{d}}$ and increasing pulsation periods ($P \simeq 6^{\mathrm{d}}$), and in Fig. 7 for Cepheids with $4^{\mathrm{d}} \leq P \leq 8^{\mathrm{d}}$ and decreasing pulsation periods ($P \simeq 6^{\mathrm{d}}$). Recall that large values of $\dot{P}$ should correspond to the hot side of the instability strip, and small values to the cool side.

The data for 20^{d} Cepheids (top portion of Fig. 5) display a tendency for large amplitude Cepheids to have rates of period increase typical of stars lying near the center of the instability strip, with smaller amplitude Cepheids falling towards the hot and cool edges (larger and smaller values of $\dot{P}$, respectively). The trend is more obvious when one plots running five-point means of the same data, as in the middle section of Fig. 5. There are two long period Cepheids with anomalously large values of $\dot{P}$, SZ Cas and AQ Pup, which are conceivably fifth crossers. If they are omitted from the running means and averages over smaller samples are included at the extremes of $\dot{P}$, one obtains the results in the lower portion of Fig. 5, which are typical of independent cross-sectional amplitude maps of the instability strip. The scatter in $\dot{P}$ values evident in the top portion of Fig. 5 is intrinsic to the stars, and is not the result of large uncertainties in the calculated values. Presumably there are intrinsic physical differences from one Cepheid to another that account for the scatter, as noted earlier. Differences in initial rotation velocity for the progenitor main-sequence stars might be the sole factor, given that they would generate sufficiently large variations in the abundances of the CNO elements throughout the star to affect the extent of the blue loop stages (Xu & Li 2004).

The data for 6^{d} Cepheids with period increases (top portion of Fig. 6) are more complicated. It appears that the sample consists of two overlapping groups of objects, a feature that also appears in the running five-point means displayed in the middle section of Fig. 6. We assume that each group consists of Cepheids displaying an order of magnitude (factor of 10) variation in $\dot{P}$ values from the hot to cool edges of the instability strip, as displayed by the long period Cepheids in Fig. 5, and use the results presented in the lower portion of Fig. 5 as a template for the likely variations in relative amplitude with $\dot{P}$ for short period Cepheids. When the two groups in Fig. 6 are separated in such fashion and averages over smaller samples are included at extreme values of $\dot{P}$ for each group, one obtains the results in the lower portion of Fig. 6. The simplest explanation for the existence of two groups among the short period Cepheids is the existence of higher strip crossings among the stars, namely fifth crossings for Cepheids undergoing period increases.

Similar results apply to the data for 6^{d} Cepheids with period decreases (Fig. 7), when analyzed in similar fashion, despite the smaller sample size. The top portion of Fig. 7 displays excessive scatter, much like that in the top portion of Fig. 6, with only marginal improvement through running five-point means (middle portion of Fig. 7). Restricting the data sets as above produces the results depicted in the lower portion of Fig. 7, which suggests an overlap between Cepheids in second and fourth crossings of the instability strip. As noted for long-period Cepheids, there is an intrinsic scatter in $\dot{P}$ values that is not the result of large uncertainties in the calculated values. Such scatter makes it difficult to use rate of period change for individual Cepheids to identify their exact location within the instability strip, although approximate placements are possible in most cases.

The conclusions reached here, while admittedly speculative, provide a reasonable explanation for the characteristic behavior of pulsation amplitude and rate of period change for individual Cepheids at specific pulsation period. As noted earlier, there may be further complications arising from the presence of overtone Cepheids in the sample, but their numbers should be relatively small in the present case, except at short periods, and they would not alter the observed distribution of data points significantly. A more comprehensive study including recognized overtone Cepheids should be possible once O–C studies have been completed for such variables.

5. Discussion

Our intent here is to demonstrate that rate of period change for a Cepheid is a useful parameter that permits one to characterize the variable in terms of specific evolutionary state. Information on $\dot{P}$ for a Cepheid, in conjunction with its known pulsation period and light amplitude, can be used to identify the strip crossing mode for the object as well as its likely location within the strip, the latter independent of its observed color and reddening. The parameter $\dot{P}$ may even be useful for establishing if a Cepheid is a fundamental mode pulsator or an overtone pulsator, although we leave that as a future exercise.

If we interpret the results of Figs. 5–7 as a generic indicator of how pulsation amplitude varies across the instability strip, then the width of the strip in $\dot{P}$ at constant period, which amounts to ~ 1.2 in log $\dot{P}$, must encompass a range of ~ 16 in $\dot{P}$. Of that, an intrinsic dispersion in log $\dot{P}$ values amounting to perhaps 0.4–0.5, a factor of ~ 3, presumably arises from actual internal differences in the Cepheids resulting from different histories for their progenitor stars. Specific stellar evolutionary models presented in Fig. 2 predict a smaller variation in log $\dot{P}$ than what is observed, which may reflect the simplicity of the models. In that regard, observed rates of period change in Cepheids can play an important role as a check on how closely stellar evolutionary models match real stars. Until now Cepheid period changes have not been used for that purpose.

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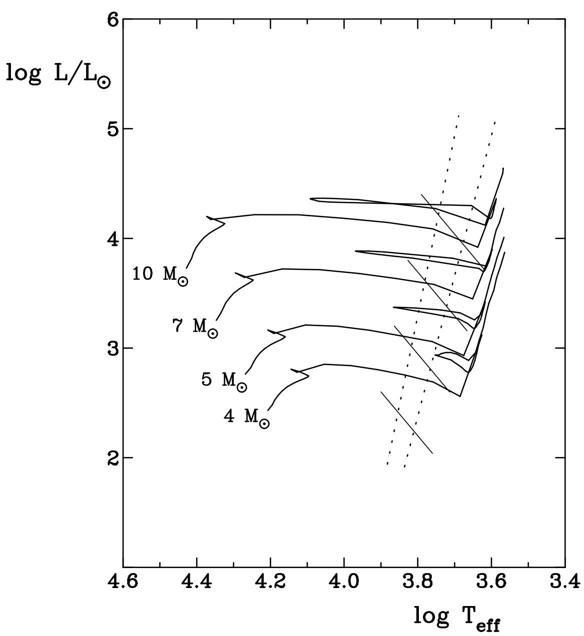


Fig. 1.— The theoretical HR diagram illustrating post-main-sequence evolutionary tracks for stars of 4, 5, 7, and 10 $M_\odot$ (Lejeune & Schaerer 2001). Included is the observational location of the Cepheid instability strip (dotted lines, from Turner 2001) and lines of constant stellar radius.

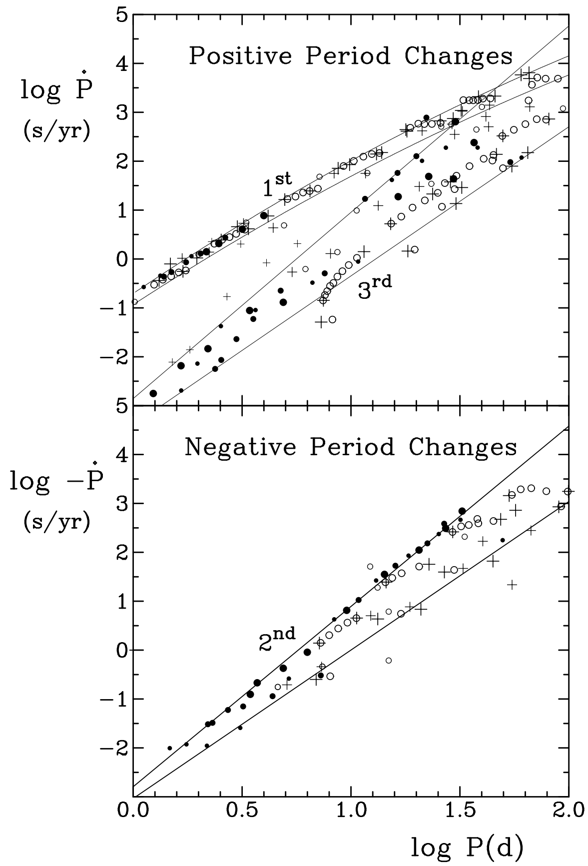


Fig. 2.— Predicted rates of period change for stars crossing the Cepheid instability strip as tied to published stellar evolutionary models. The meaning of the different symbols is explained in the text. Lines denote regions within which the predictions from different stellar evolutionary models appear to cluster. The different crossings of the instability strip are identified.

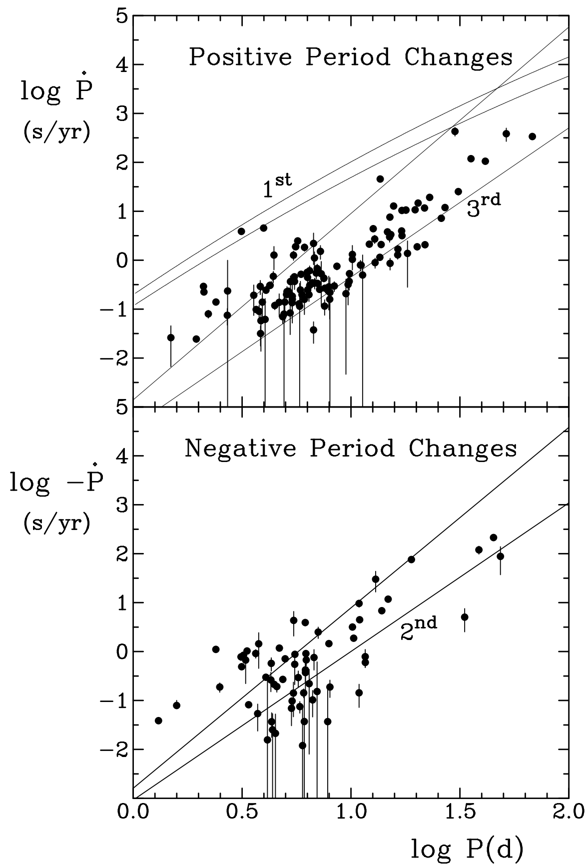


Fig. 3.— Observed rates of period change, along with their calculated uncertainties, for well-studied Cepheids possessing many years of O–C data. Dotted lines are the relations depicted in Fig. 3, and the different strip crossings are identified.

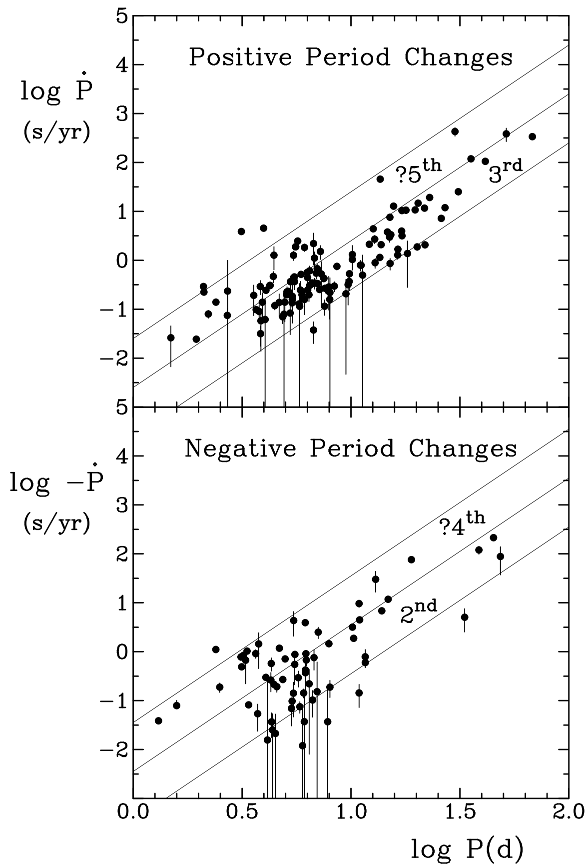


Fig. 4.— The data of Fig. 3 plotted along with suggested empirical delineations of the regions corresponding to Cepheids in different crossings of the instability strip, as identified.

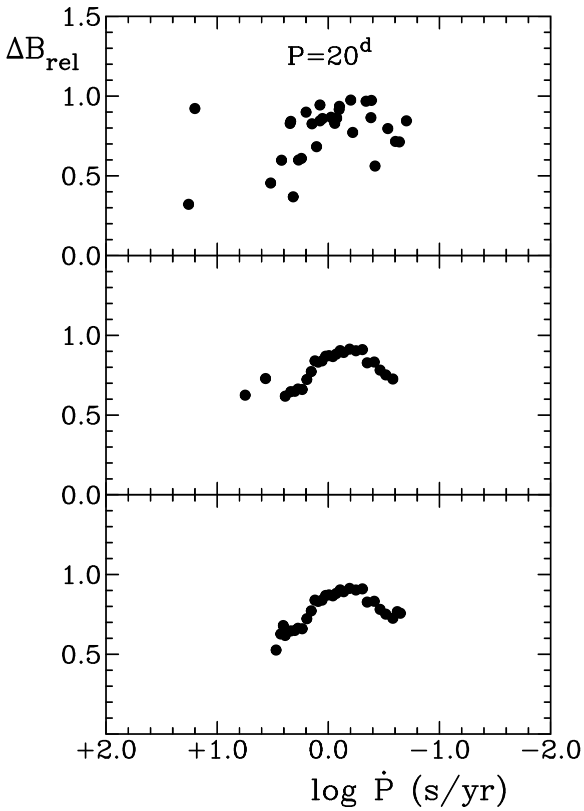


Fig. 5.— Normalized blue amplitudes of Cepheids with $12^{d} \leq P \leq 40^{d}$ and increasing pulsation periods as a function of normalized rate of period change (upper section). The middle section displays running five-point means for the data, and the lower section adjusted and extended means of the data.

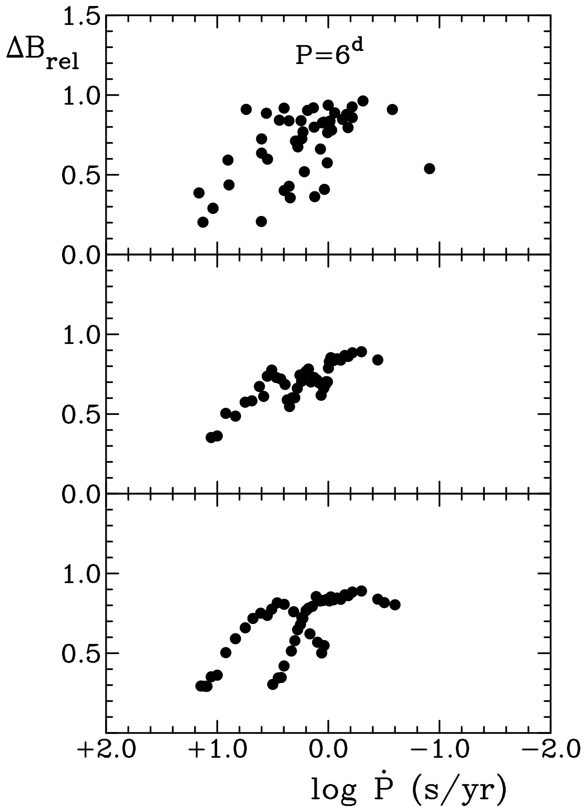


Fig. 6.— Normalized blue amplitudes of Cepheids with $4^{\mathrm{d}} \leq P \leq 8^{\mathrm{d}}$ and increasing pulsation periods as a function of normalized rate of period change (upper section). The middle section displays running five-point means for the data, and the lower section adjusted and extended means of the data after judicious separation of the overlapping samples using the results of Fig. 5 as a template.

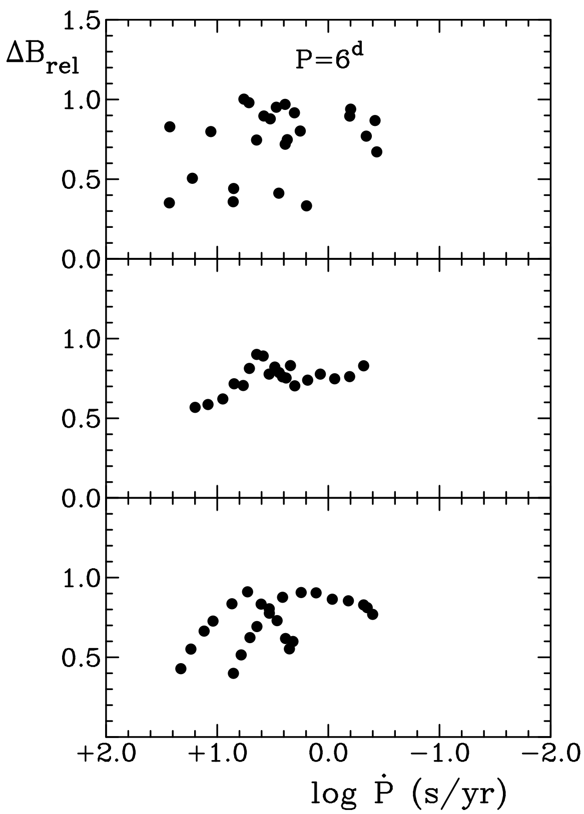
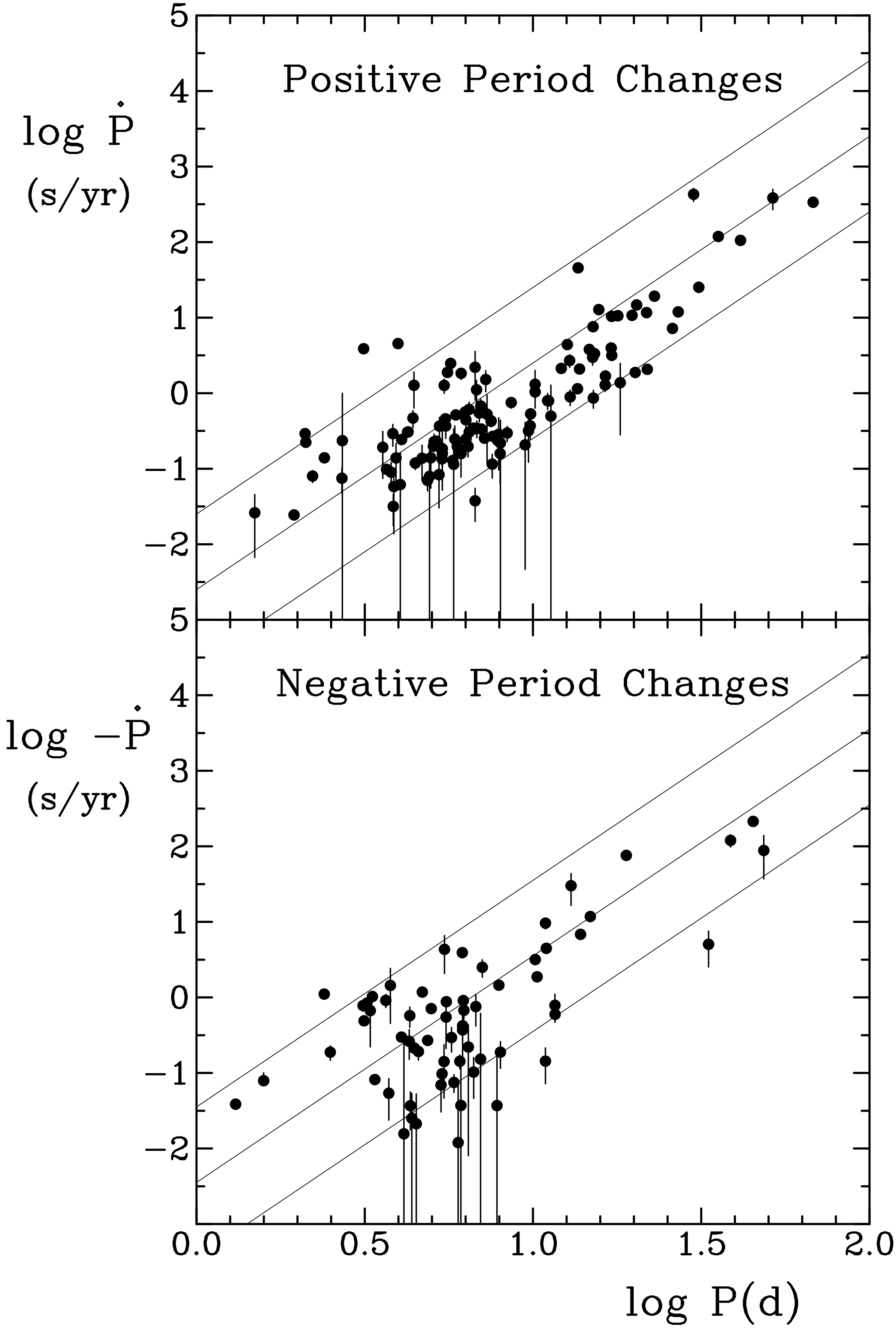


Fig. 7.— Normalized blue amplitudes of Cepheids with $4^d \leq P \leq 8^d$ and decreasing pulsation periods as a function of normalized rate of period change (upper section). The middle section displays running five-point means for the data, and the lower section adjusted and extended means of the data after judicious separation of the overlapping samples as in Fig. 6.


Positive Period Changes
log $\dot{P}$
(s/yr)
Negative Period Changes
log $-\dot{P}$
(s/yr)
log P(d)
5
4
3
2
1
0
−1
−2
5
4
3
2
1
0
−1
−2
0.0
0.5
1.0
1.5
2.0